Building Typologies of Individual Trajectories: An Overview of Statistical Methodologies

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Plan

- 1. Reminders about clustering methods
- 2. Specificities of longitudinal data
- 3. Typologies of pathways
- 4. An example

- Conclusion

1- Reminders about clustering methods

Goals and general principles

- **Clustering** is the classification of objects into different groups (partitioning into clusters).
- A way to **explore** a data set, particularly adapted to the case of complex and numerous data.
- « Discover » (or confirm) some underlying pattern: the existence of groups (in case of heterogenous data); or create instrumental classes.

  Methods may refer to a notion of proximity, to a model, to a notion of density. We will focus on the first approach: « Gather what is alike »

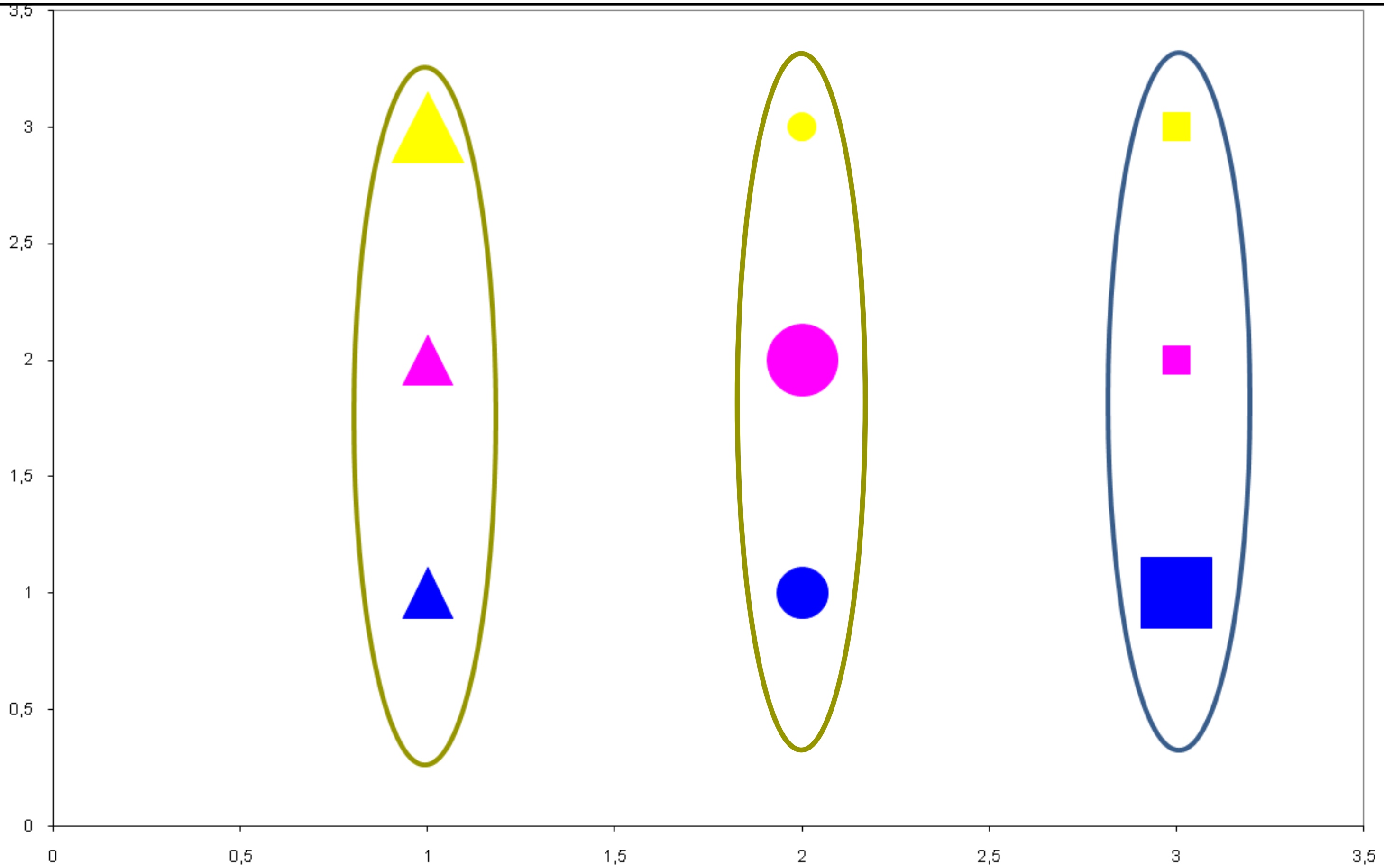
4 main steps

1. Data preparation: selection, coding and transformation, organisation
2. Dissimilarity measure between objects
3. Clustering method (measure of dissimilarity between sets of objects)
4. Consolidate the results; interpret them

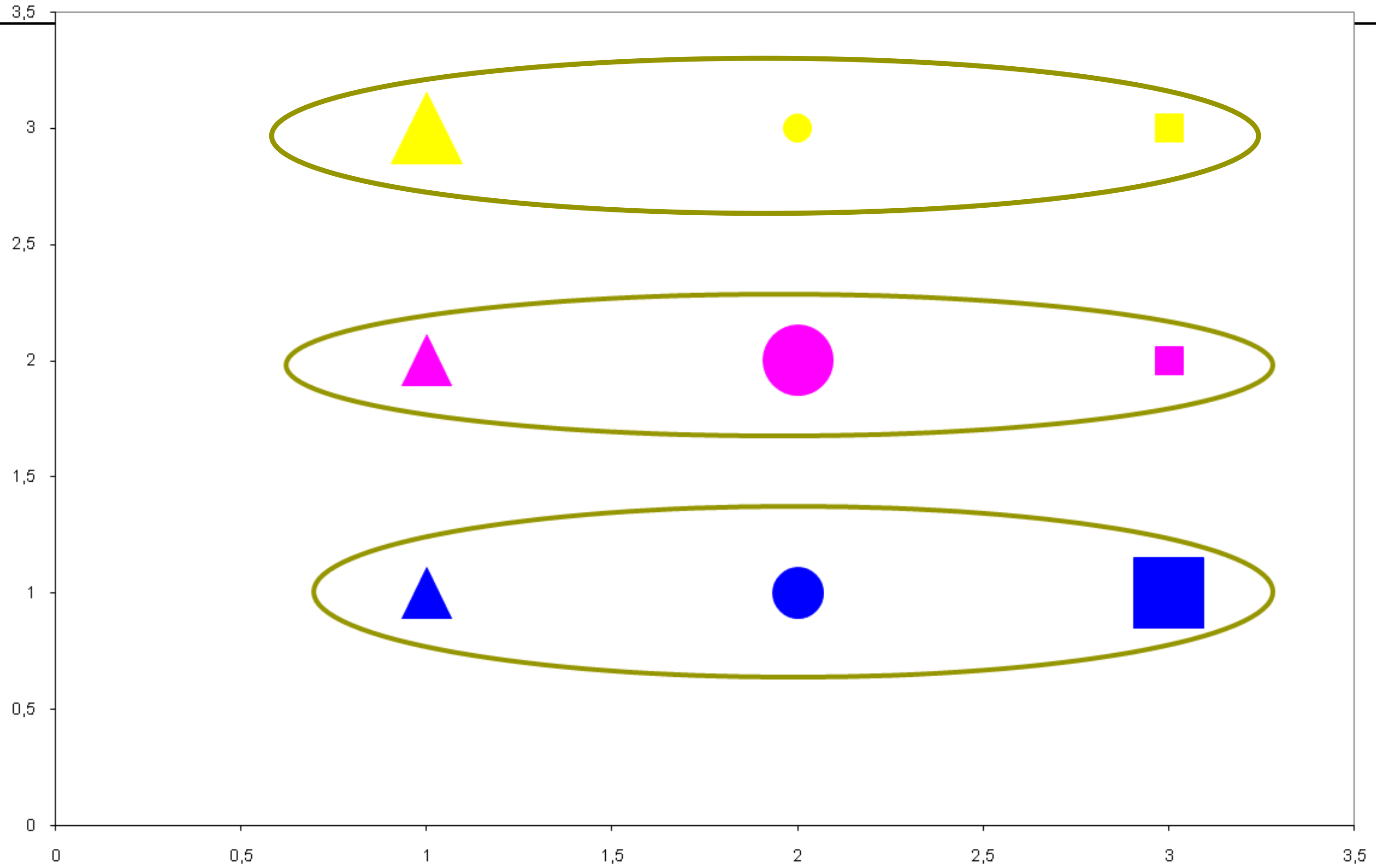
1.1. Data preparation: selection, coding, transformation, organization

- Data selection
  - Individuals (elimination of outliers)
  - Variables (Homogeneity of data)

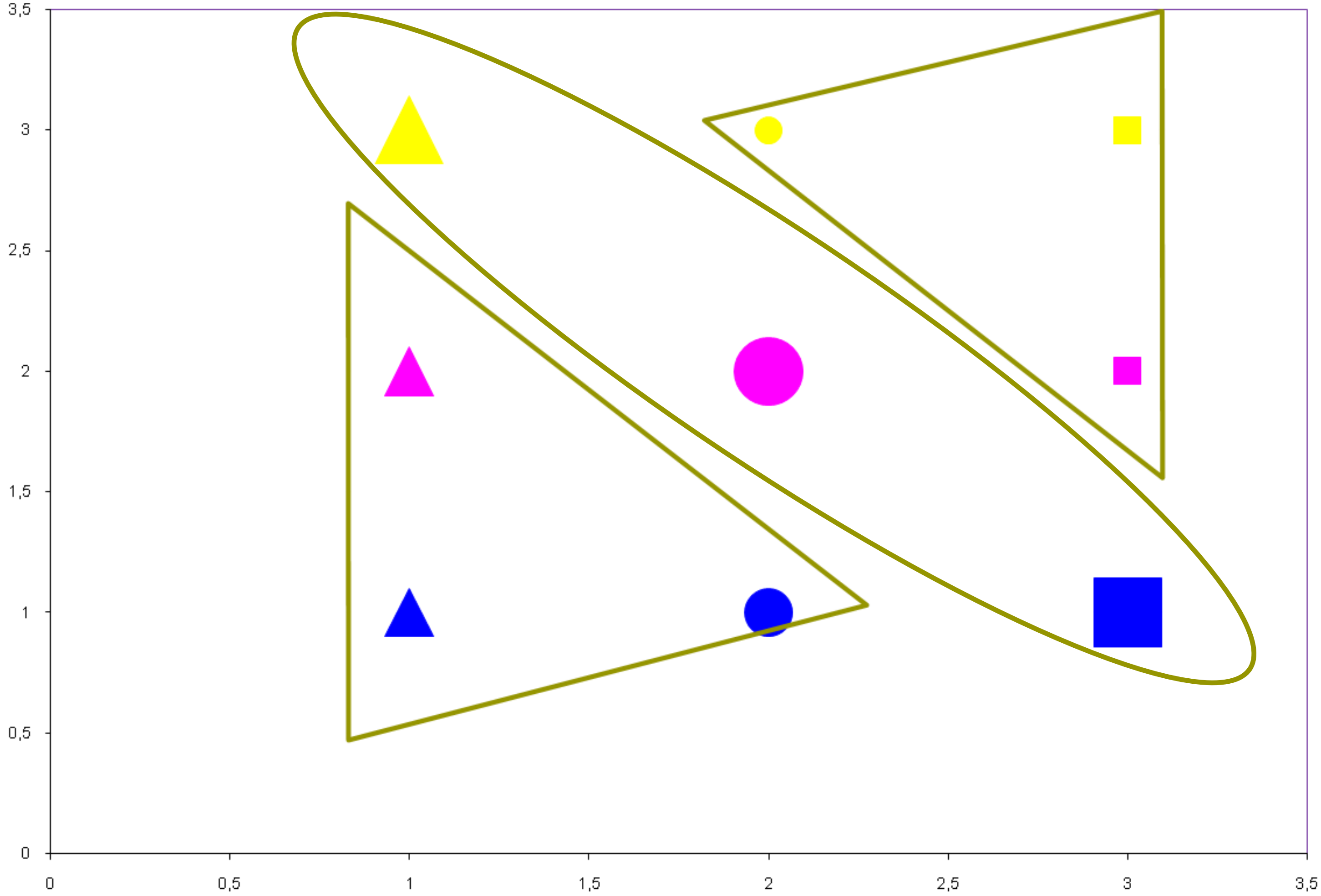
1.1.1. Data preparation: selection of variables. On which attribute are things alike? (1)



On which attribute are things alike? (2)



On which attribute are things alike? (3)



1.1.2. Data preparation: coding, transformation, organization (1)

- Coding, transformation
  - Quanti + quali → quali (homogeneisation)
  - Imputation of missing data…
- Organization
  - Table with I rows (individuals =observations) and J columns (variables)
  - If distance matrix → coordinates

1.1.2. Data preparation (2)

- Highly recommended, if possible (often) : to perform a factor analysis (CA or PCA) before Clustering, in order to:
  - Plot the data on the factorial planes and see the shape of the cloud (well separated clusters or continuum?)
  - Detect outliers
  - Perform the clustering on the first components (data transformation which eliminates noise, makes data homogeneous)

1.2. Measure of likeness between observations

- Quantitative data: a distance

  Euclidean distance: $D^2(i,i') = \Sigma_j \, w_j \, (X_{ij} - X_{i'j})^2$

  Where w_j is the weight of variable j.

- Qualitative data: dissimilarity coefficient based for example on the number of concordances and discordances (a great variety of such indexes).

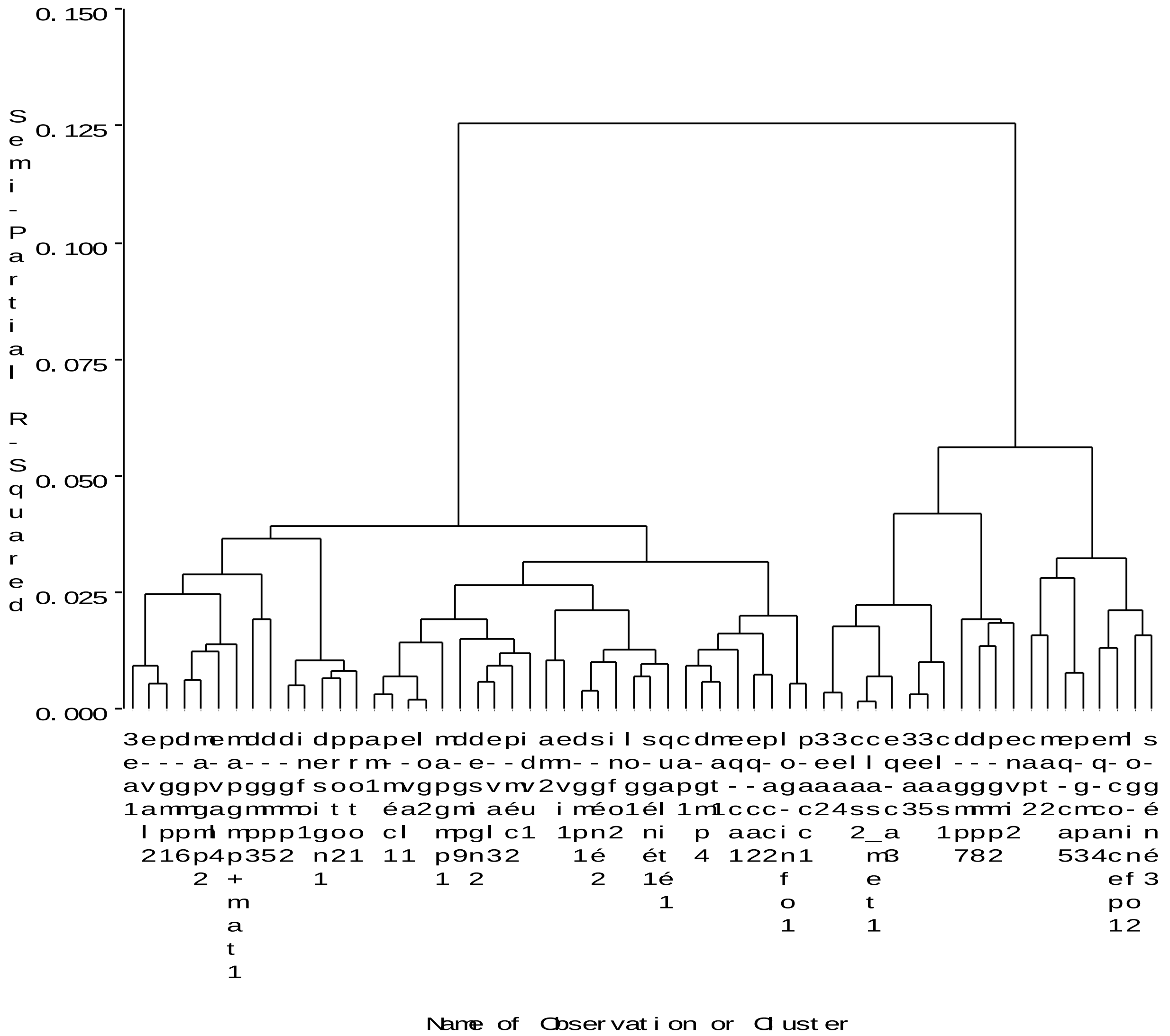
1.3. Choice of a clustering procedure

- Two groups:
  - Hierarchical clustering
  - Partitioning
- And neural networks

1.3.1. Hierarchical clustering

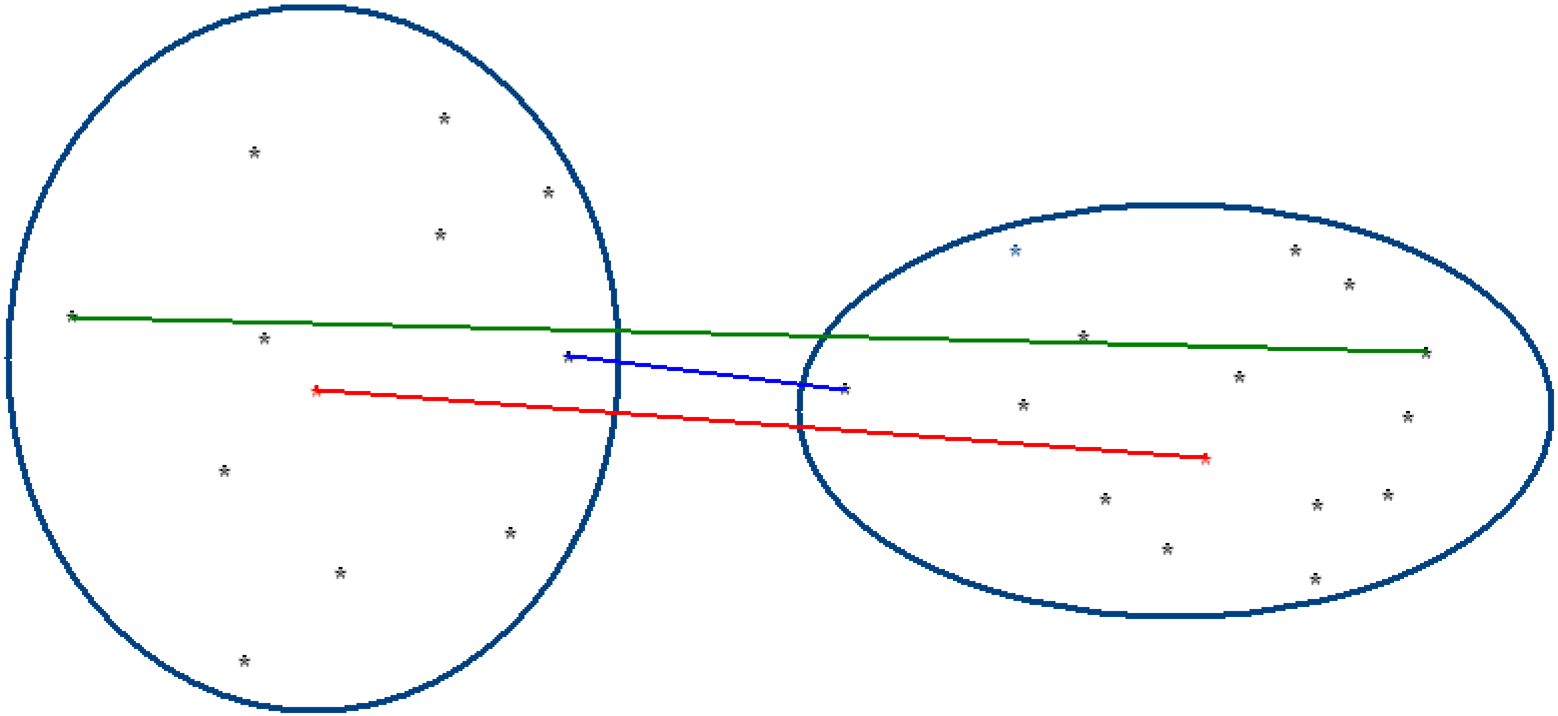
- At each step
  - Two clusters are agglomerated (hierarchical ascending classification - HAC)
  - Or one cluster is split into two smaller clusters (descending classification)

  Clusters form a tree structure (dendrogram)


Semi-Partial R-Squared
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Name of Observation or Cluster

Examples of linkage rules

Single linkage, Complete linkage, Between centroids, Group average



A very usual agglomerative criterion: Ward’s

- Ward’s criterion: maximize variance between (minimize within) groups

A general frame: « Flexible » distances

- Lance and Williams formula

  $d(C, A \cup B) =$

  $\alpha_A \, d(C,A) + \alpha_B \, d(C,B) + \beta \, d(A,B) + \gamma \, |d(C,A) - d(C,B)|$

  with $\alpha_A + \alpha_B = 1; \beta < 1; |\gamma| < 1$

  Examples:

  - Single linkage: $\alpha_A = \alpha_B = 1/2$
  - Complete, Centroid, Median, Group average, Ward,
  - Flexible beta…

1.3.2. K-means partitioning

- Number k of clusters set *a priori*
- k seeds are selected as first guess of the means
- Observations agglomerated according to their *distance* to the seed, into k clusters
- Agglomeration around the centroid of the clusters
- Aso until convergence
- Possibly start over again with other sets of k seeds, to obtain several classifications (sets of k disjoint clusters). Crossed classif. gives some stable clusters.

1.3.3. Hybrid clustering (Wong-Lebart)

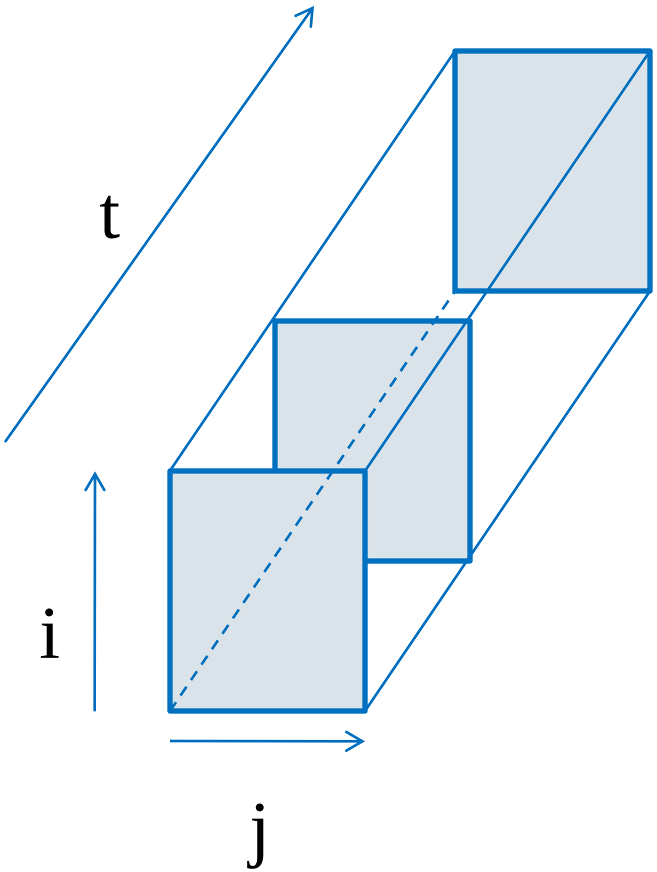
- Preliminary clustering with a k-means method to search stable groups, which may be numerous
- Hierarchical clustering of the previously obtained clusters, cutting of the dendrogram: determination of the final number of clusters k
- The centers of the previous clusters are taken as seeds for a partitioning procedure in order to consolidate the clusters

1.4 Interpretation of results

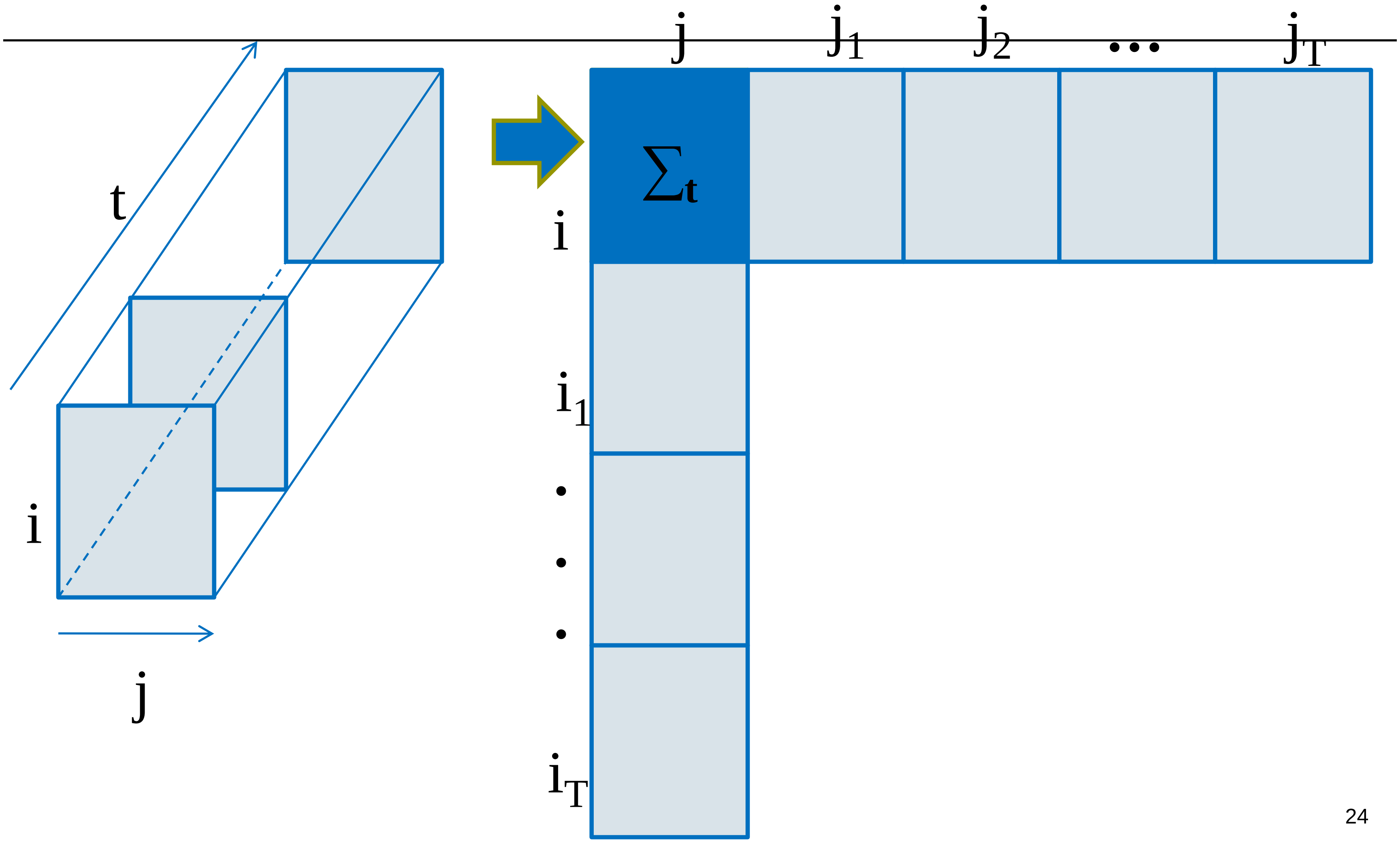
- Compute the average profile of clusters (active variables, individual characteristics)
- Factor analysis combined with cluster analysis after clustering, in order to:
  - See the position of the clusters and their centroid on the factorial planes → interpretation of clusters/factors
  - Characterize the clusters
- Display the representative elements of each cluster (the type)
  - Fictious modal element of each cluster
  - The real element(s) which are near from the centre of gravity of the cluster

2- Longitudinal data

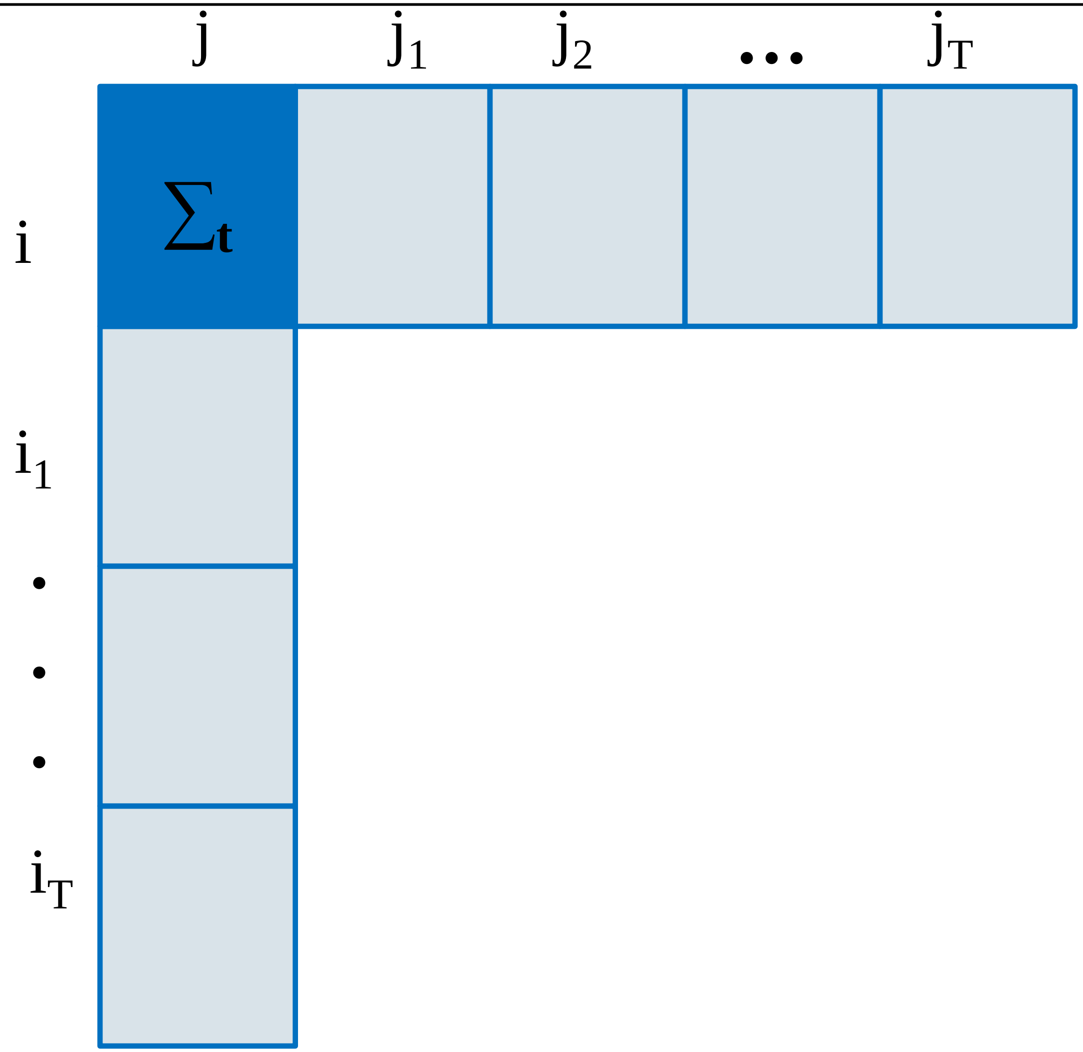
2.1. General frame: Three-way data table

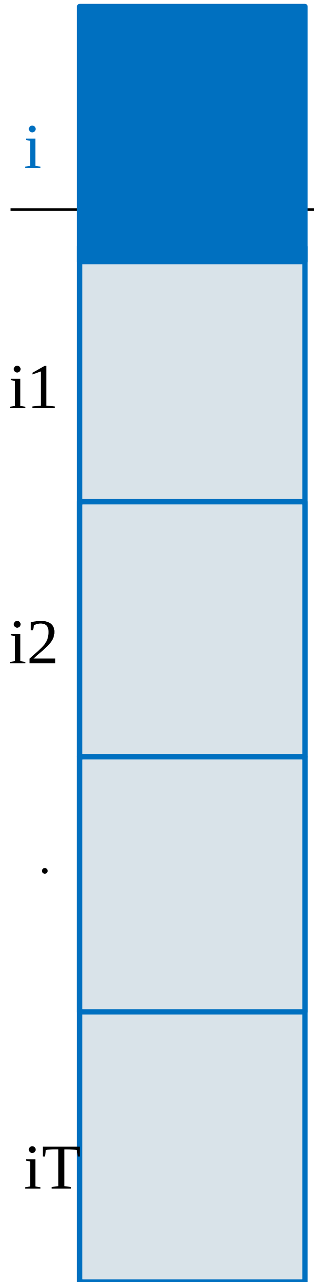


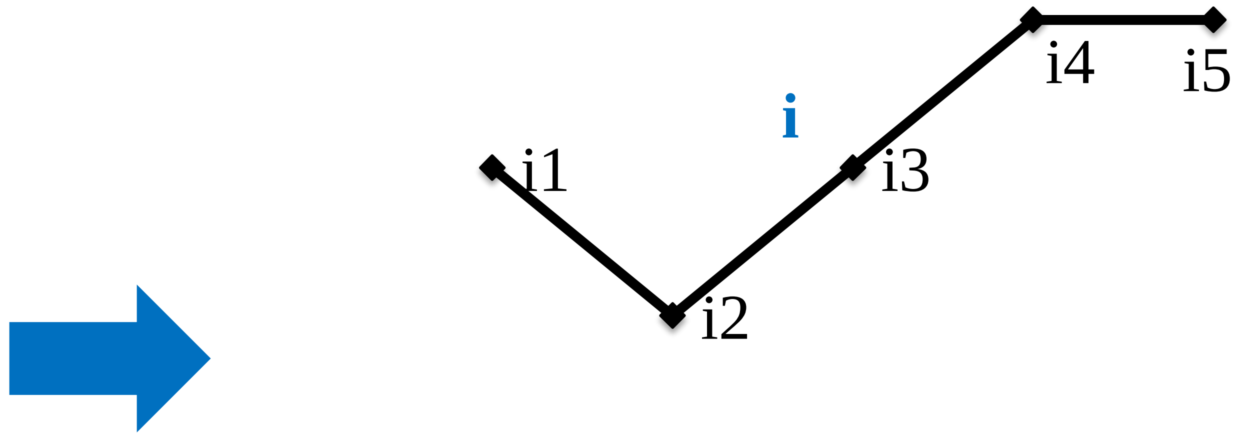
The 3-way table and its (un)foldings



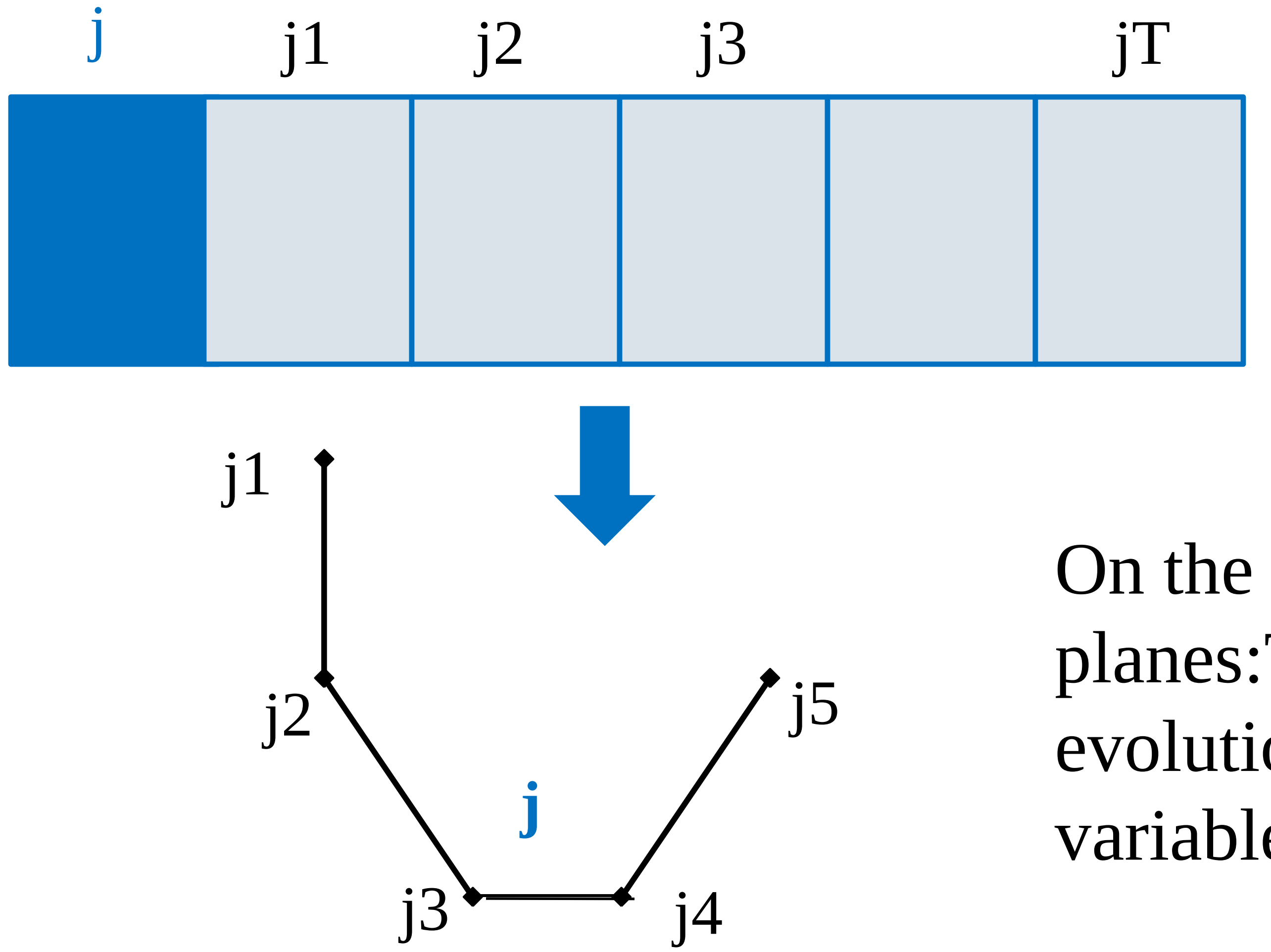
Factor Analysis of 2-way tables







On the factorial planes:
Trajectories of individuals
(or groups of individuals)



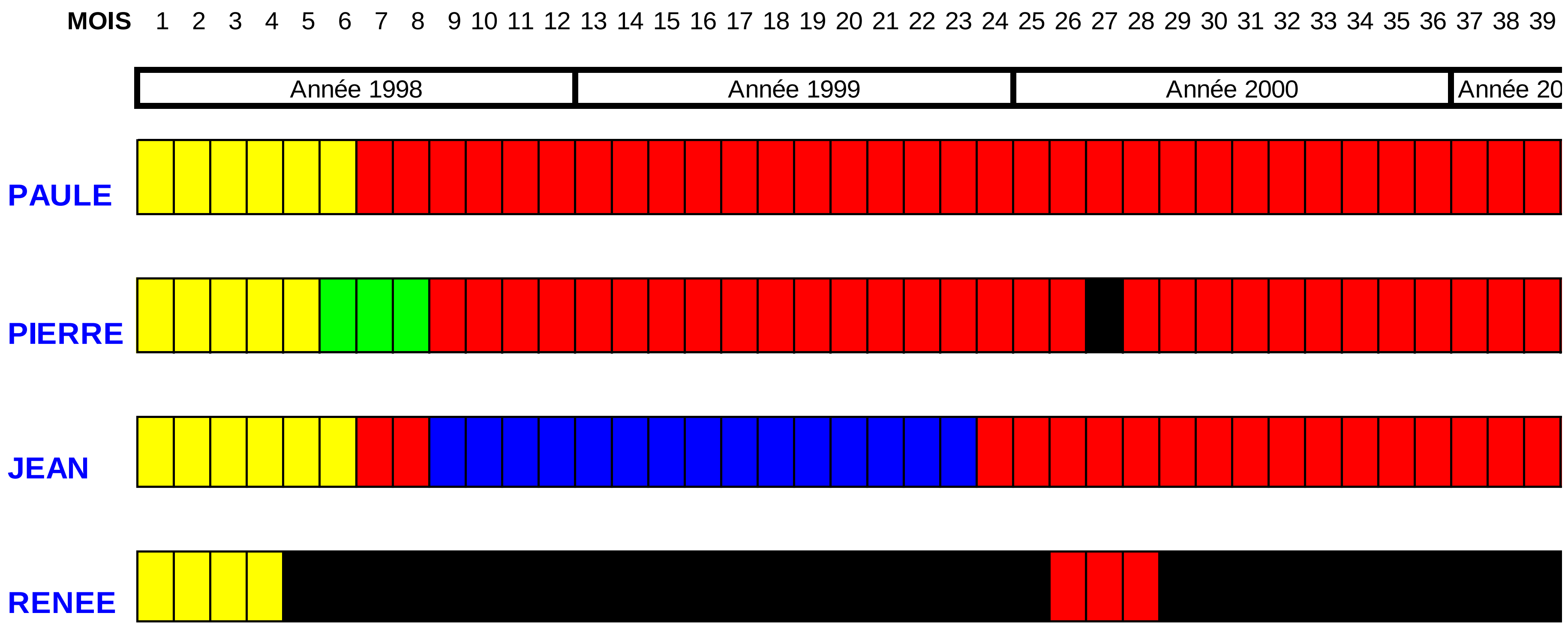
On the factorial planes:Time-evolution of variables

2.2. Example

- Céreq's survey on transition from school-to-work

Céreq's « Generation 98 »' monthly calendar

State | **At school** | **Employed** | **Unempl** | **National Service** | **Other**

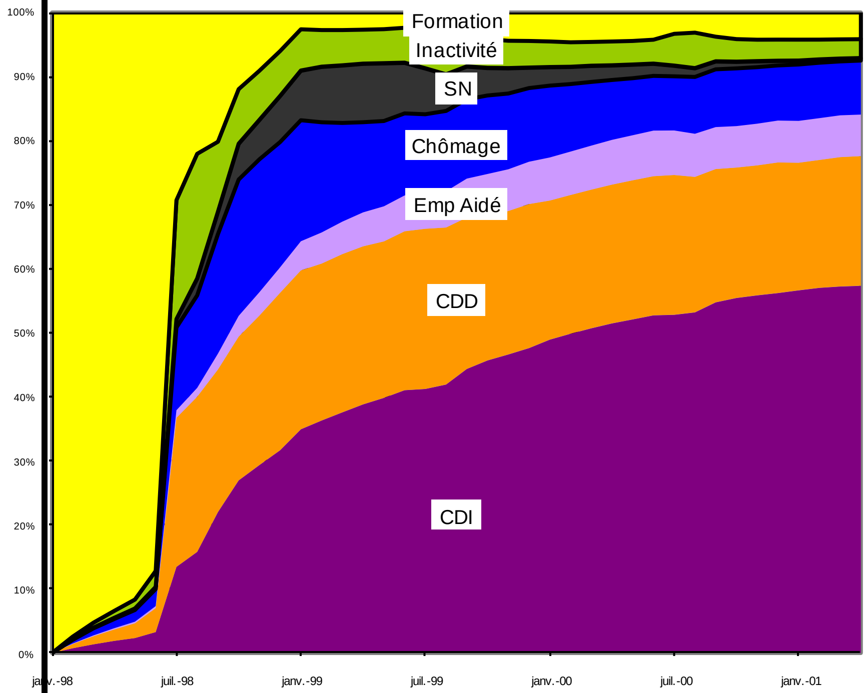
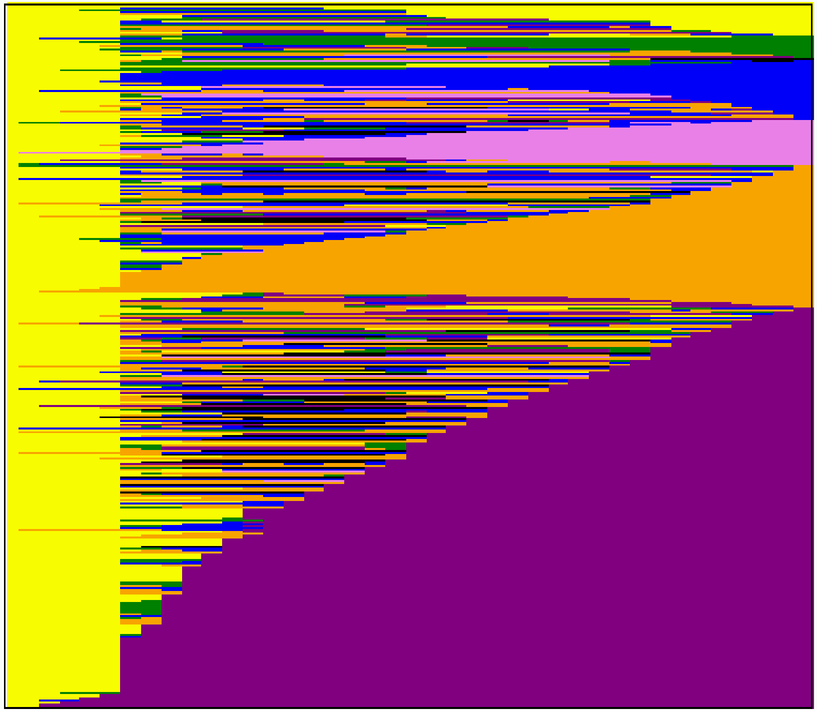


Display of individual transition pathways

Two types of graphs:

- Chronograms
- « Carpets»

Monthly histogram / Individual pathways

Generation 98'Cohort	Generation 98' pathways
	

- **3- Typologies of pathways**

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Goals

Explore the diversity of pathways:

ex) 5 states, 12 months → 5^{12} (>200M) possible pathways

Identify clusters of « similar » observations, types of transition pathways

Summarize pathways:

Compute a new variable, which may be used in a model

- Factor analysis → interval variable
- Cluster analysis → nominal variable

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A series of decisions

- **Upstream** from clustering procedure:
  - How describe pathways?
    - Selection of the relevant information (which states?)
    - Coding of the information (monthly calendar, indicators, …)
  - How measure dissimilarity (choice of a distance)
- Choice of a **clustering method** (agglomerative procedure)
- **Downstream**: understand why observations are in the same cluster (characterize clusters)

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Coding Monthly Calendars

- Dummy coding restoring the whole information:
  S=nbr of states, M=nbr of Months → SM variables x_{sm} with value 1 if observed state s on month m, 0 otherwise.
- Transitions between states (S^2 values)
- Both
- Summarized calendar = Qualitative Harmonic Analysis
- More summarized: Indicators
  ex) time spent (or %) in each state, nbr of spells for each state, duration before access to a particular state, etc.

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Choice of a dissimilarity index

- Depends on the nature of variables resulting from the coding (quali/quanti)

- Examples with monthly calendars:
  - Euclidean distance (discordance)
    $D_L^2(i,i')=\Sigma_m \delta^{i'm}_{im}$ where $\delta^{i'm}_{im}=1$ if i and i' are in different states during month m, 0 otherwise
  - Chi-square distance
    (row i, column sm, dummy coding)
    $D_C^2(i,i')=(1/M)\ \Sigma_{sm}\ (1/f_{sm})\ (k_{ism}-k_{i'sm})^2$

To weight or not to weight distances

- Why? Take account of:
  - Proximity between states
  - Frequency of states' occurrence
  - Time (proximity and frequency may vary over time)

Qualitative Harmonic Analysis

- Deville et Saporta (1980)
- Method to describe « a set of individuals characterized by a career, that is (to say) by an evolution in a finite set of states» (Deville, 1982)
- Little literature using this method: Barbary (1993), Degenne *et al* (1995), Grelet (2002)

QHA: methodology

- Choice of a common period of observation and division of the period into intervals
- Measuring, for each interval, the proportion of time spent in each state
- Factor analysis of the created variables
- Cluster Analysis on the factors

➔ Typology of trajectories

Split of the period

- Choice of the observation period:
  - The same for all individuals
  - Censoring is not taken into account

- How to split the period:
  - Split into intervals of equal durations,
  - Split according to events quantiles,
  - Arbitrary split…

- Number of intervals:
  - If too high: many variables equal 0
    - ➔ sparse matrix and bad quality factor analysis
  - If too low: important reduction of information
    - ➔ bad quality description

Coding the variables

- Choice of states nature and number:
  - Same trade-off as for number of intervals

- Computation of the proportion of interval duration spent in a state

  ➔ nb of variables = nb of states X nb of intervals

- Possible refinement (Degenne *et al*, 1995):
  - Addition of series of variables calculating the proportion of each transition between two states

    ➔ nb of variables = (nb of states)^2

Example

Trajectory = A B B B A

- 2 states: A and B
- Period: t1 to t5, split into 2 intervals: [t1;t2] and [t3;t5]

➔ 4 + 4 variables :

interval [t1; t2]		interval [t3; t5]		transitions			
A	**B**	**A**	**B**	**AB**	**AA**	**BB**	**BA**
0,5	*0,5*	*0,33*	*0,66*	*0,25*	*0*	*0,5*	*0,25*

Factor analysis

- Correspondence Analysis on the variables
- Only a part of the factors are retained, so as to discard « noise » while keeping the major part of information (70% to 80% of inertia)
- Hierarchical Cluster Analysis on selected factors

Description of trajectories by means of QHA

3 dimensions compose a trajectory:

- DURATION spent in each state: *measured by duration variables*
- MOMENT when one's in each state: *introduced by the split into intervals*
- TRANSITIONS from a state to another: *measured by transition variables*

QHA limits

- Problem of the censorship not taken into account
- Sequence of states not taken into account

Sequence analysis

- Individual trajectories are built as sequences of states

Example: a school-to-work trajectory (8 months):

S = student; U = unemployed; W = wage-earner

m1	m2	m3	m4	m5	m6	m7	m8
S	S	S	U	W	W	W	W

- Then they are grouped together according to their degree of similarity

  technique = *optimal matching analysis (OMA),* …

➔ Typology of trajectories

Optimal Matching Analysis (1)

- Method born from molecular biology (DNA strings)
- Introduced in social sciences by Andrew Abbott in the 80's
- ***Principle***: measuring dissimilarity between pairs of sequences by calculating the cost of the transformation of one sequence into the other

see e.g. (Abbott & Tsay, 2000)

Optimal Matching Analysis (2)

- 3 basic operations:

  - insertion
  - deletion
  - substitution

- Each operation is assigned a cost

- The distance between two sequences is equal to the minimal cost needed to transform one sequence into the other

- A cluster analysis of the distance matrix (comparison of all pairs of sequences) gives typologies

Optimal Matching Analysis (3)

Example:

X: B B A B A B

Y: B A B A B B

X: B B A B A B

Y: B **B A B A** B

→ *4 substitutions*

X: B B A B A B

Y: **B** B A B A B ~~B~~

→ *1 insertion, 1 deletion*

The choice of costs (1)

- Crucial issue of OMA: sequence = event + time

- *Substitution*: retains the temporal structure (moment) but distorts events structure (order)

  Insertion/deletion: distorts time but retains the order of events

- A common choice: indel=1, subst=2

The choice of costs (2)

- Substitution costs:
  - According to theoretical assumptions:
    → *ex) stratification*
  - Data driven:
    → *ex) based on transition likelihoods*

Criticism

- Different lengths: censoring or process?
  - ➔ Variable indel costs (Stovel, Bolan, 2004)
- Substitution does not take order into account
  - ➔ Common sub-sequences (Dijkstra, Taris, 1995; Elzinga, 2003)
- Trajectories are time-dependent
  - ➔ No indel operations, substitution costs computed for each moment (Lesnard, 2004)

Conclusion

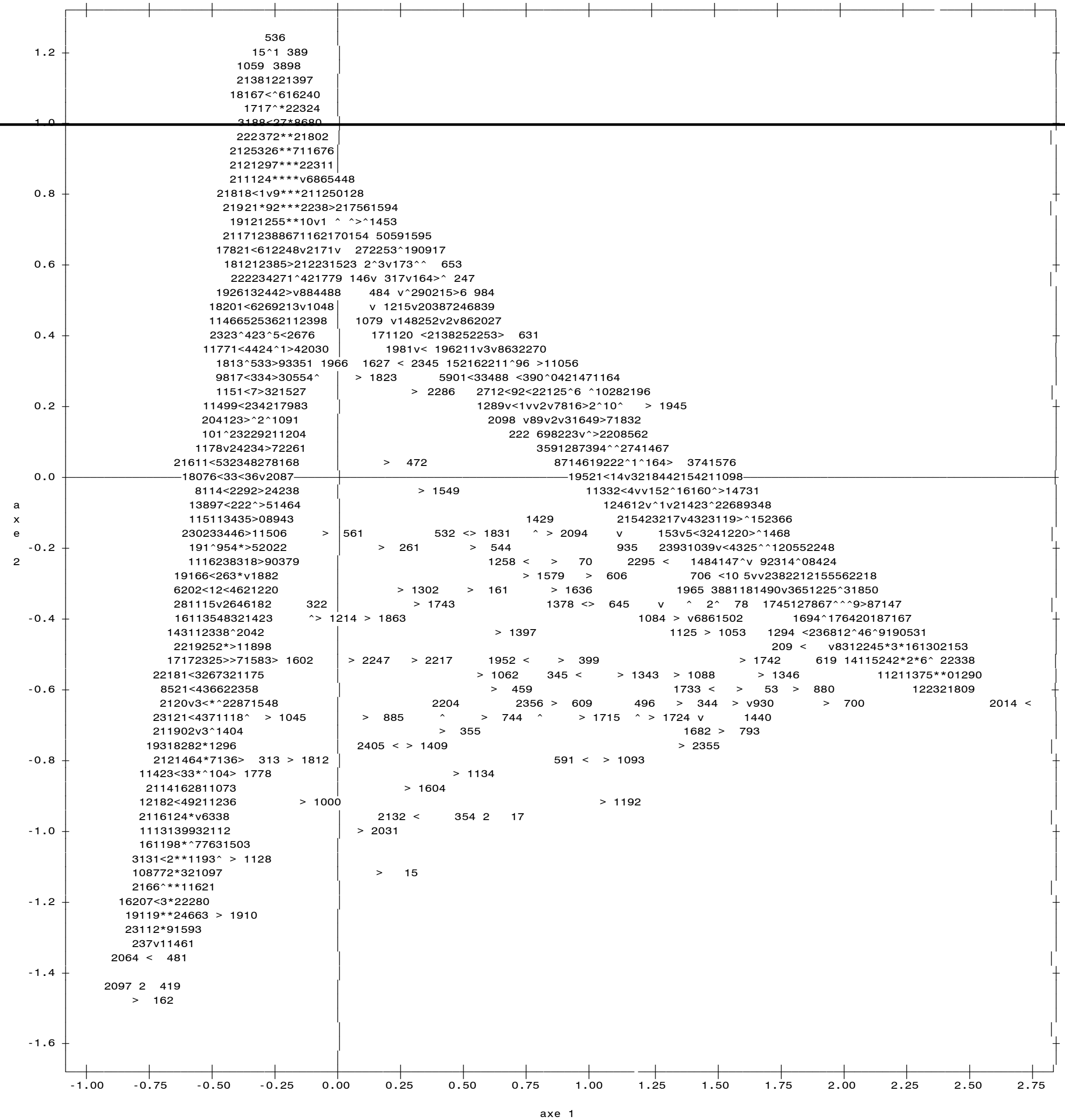
- exploration of complex trajectories
- complementarity with stochastic approaches
- robustness
- flexibility
- reflection about data and research question:

  through coding; costs (OMA)…
- OMA: possible varying length of pathways (but… the sense of it?)

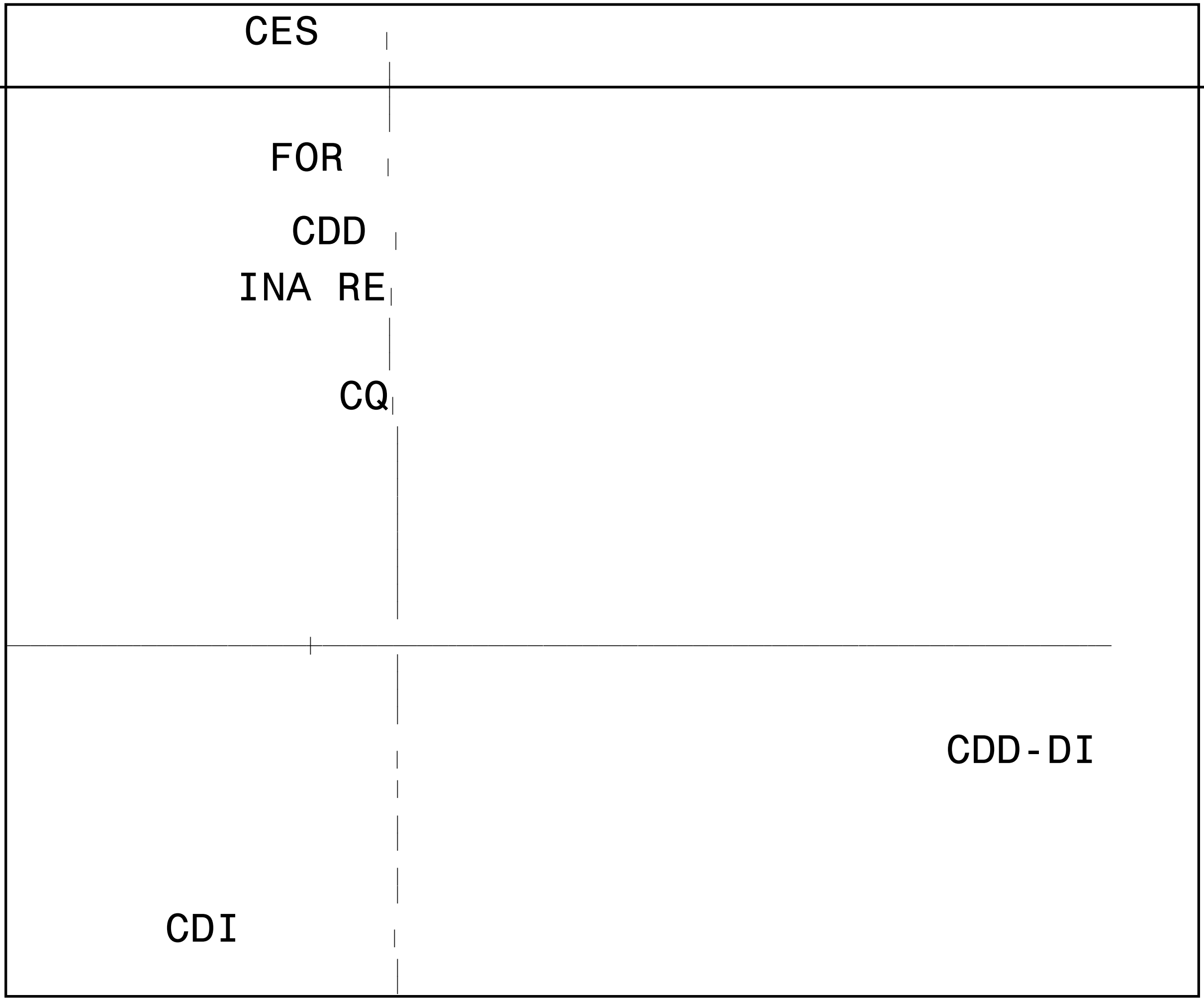
- **4- EXAMPLE**

- 2400 young people having exited school in 1992, surveyed in 1997
- 64 Months' calendar
- 8 States:
  - Unlimited term contract (CDI)
  - Limited term contract (CDD)
  - Stabilisation from unstable to stable contract (CDD-DI)
  - Employment and training scheme (CQ)
  - Part-time Employment scheme without training (CES)
  - Unemployment (RE)
  - Inactivity (INA)
  - Return to education, training (FOR)
- MCA of the table: 2400 rows x 512 columns (8x64)
- Total time spent in each state as supplementary variables

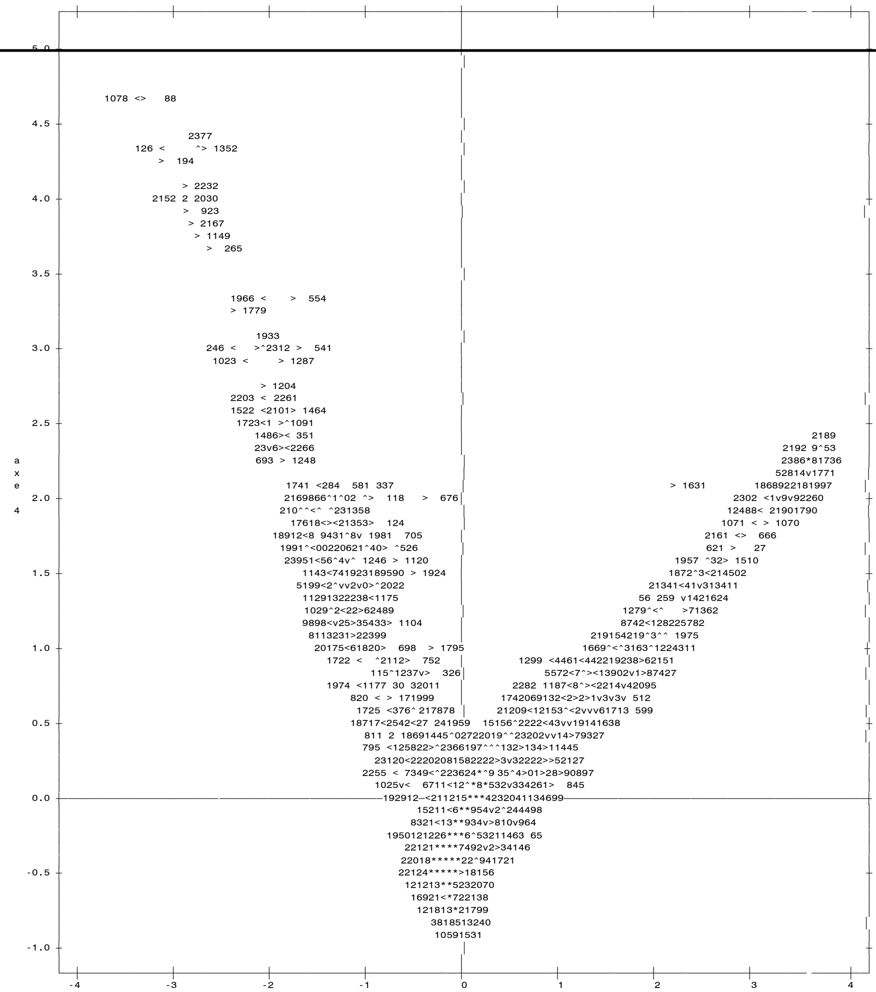
MCA – Plot F1xF2, individuals



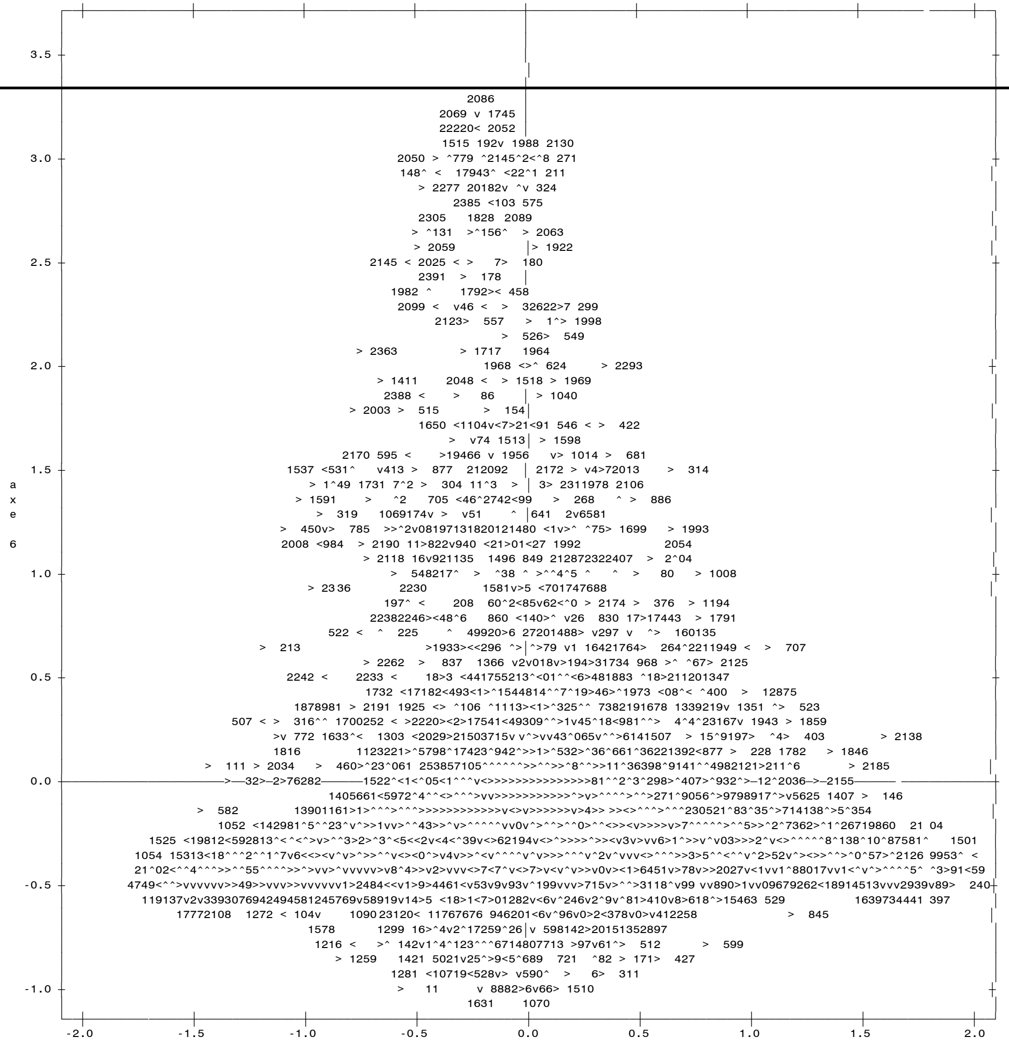
MCA – Plot F1xF2, 8 states (nbr of months)



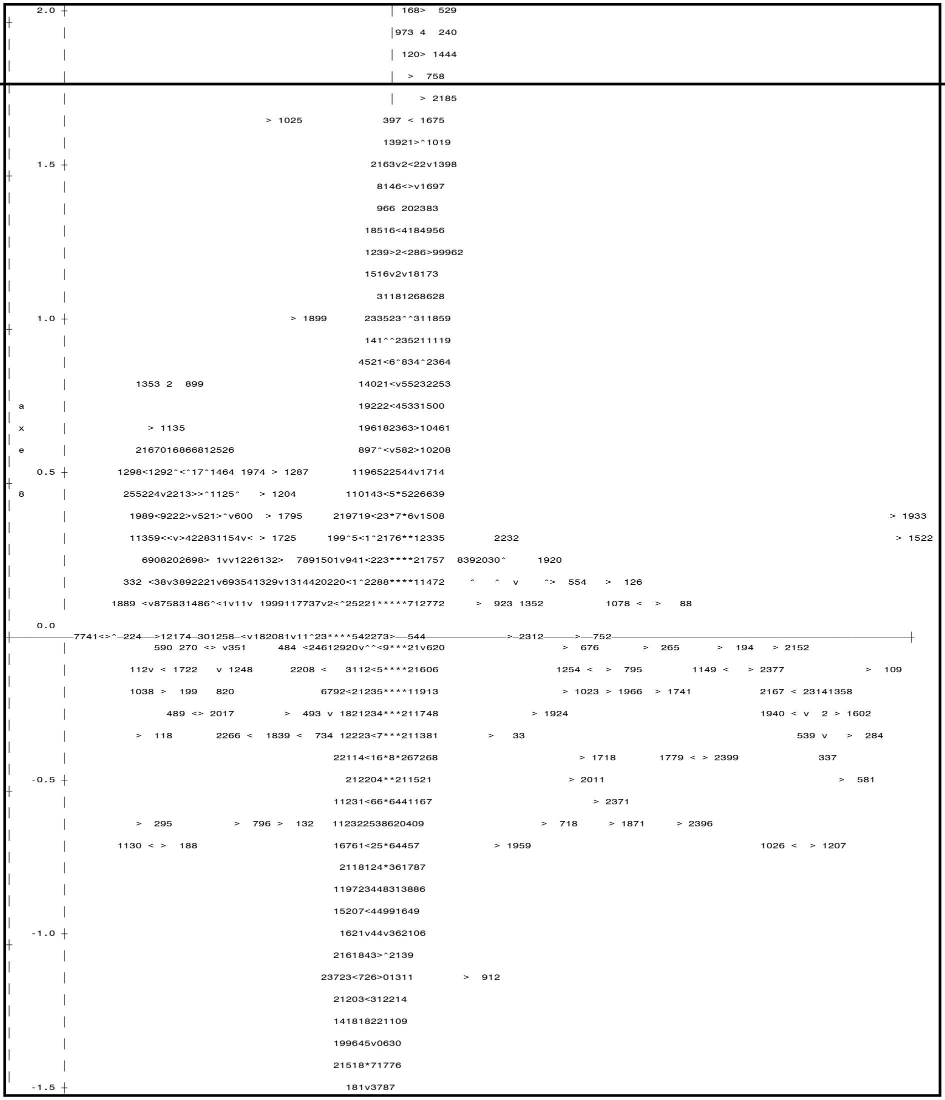
MCA – Plot F3xF4, individuals



MCA – Plot F5xF6, individuals



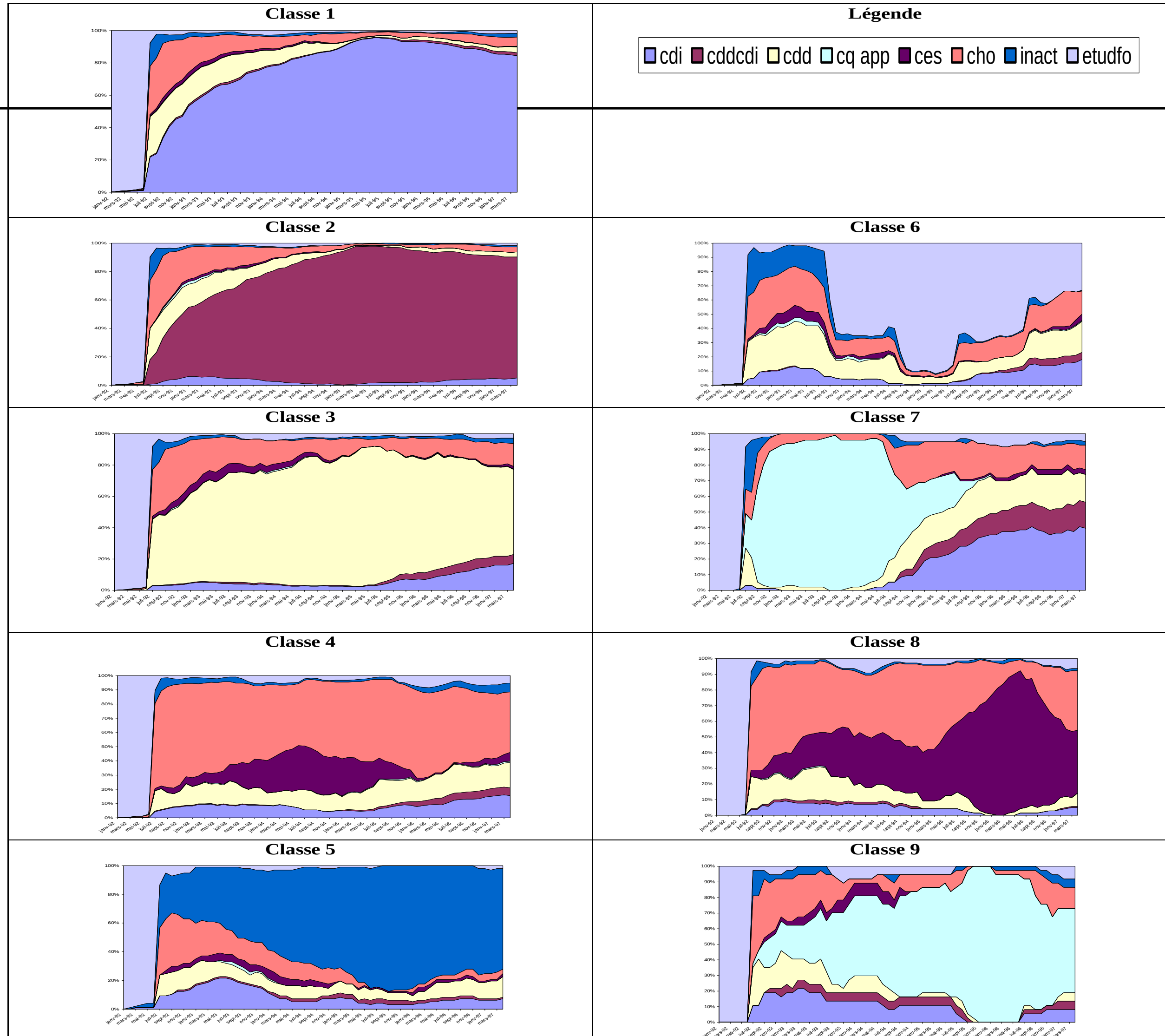
MCA – Plot F7xF8, individuals



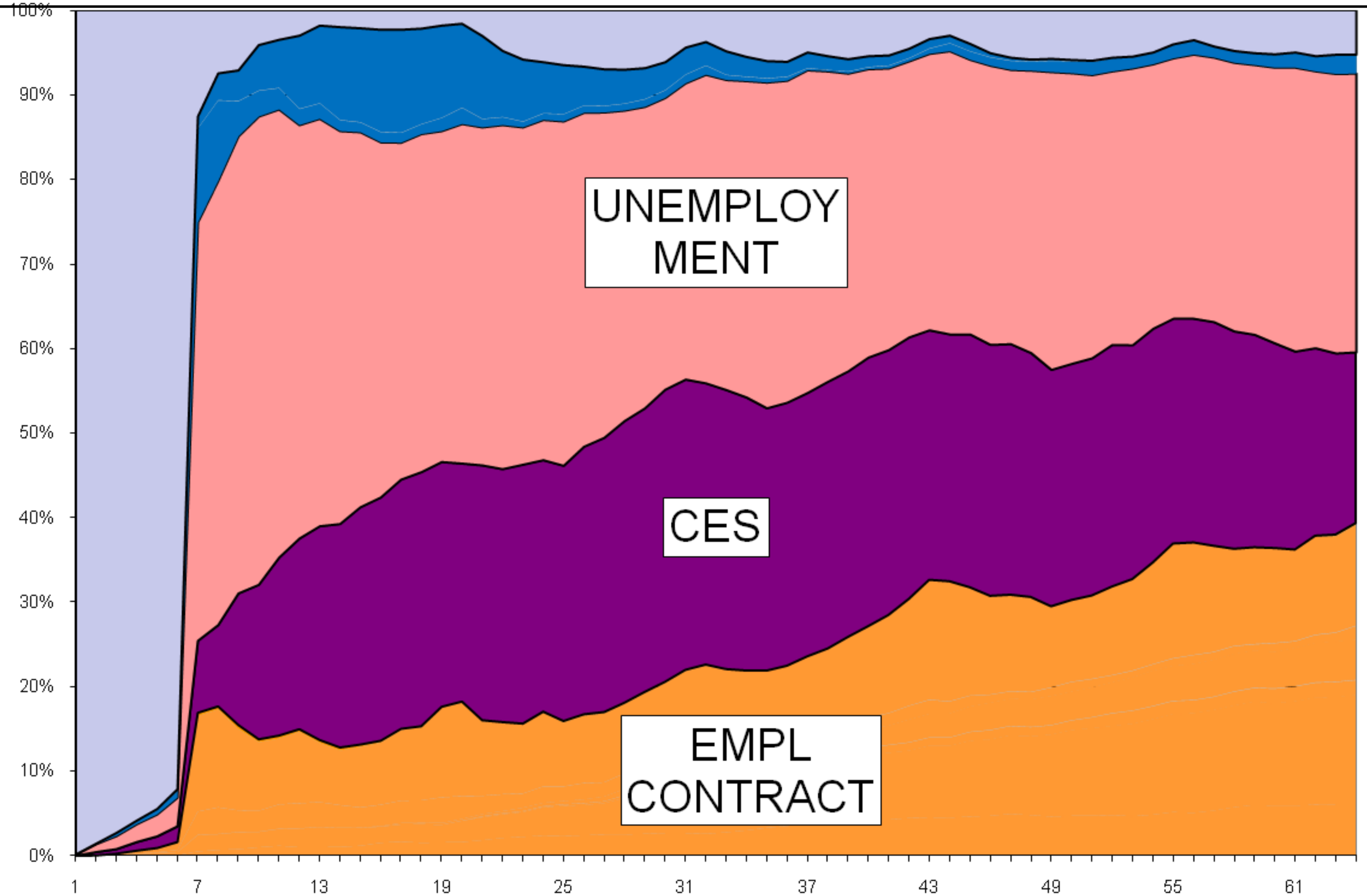
MCA Outcomes

- The successive 2-dimensional plots show that:
  - There is no evident clump of individuals, but rather a continuum, at least on the first axis
  - The main differentiation between pathways is due to occurrence of states: did youngsters experience unemployment -stable contract, inactivity, etc. - or not?
  - Time variation is minor compared to « synchronic pattern » (appears only after the 6th factor)

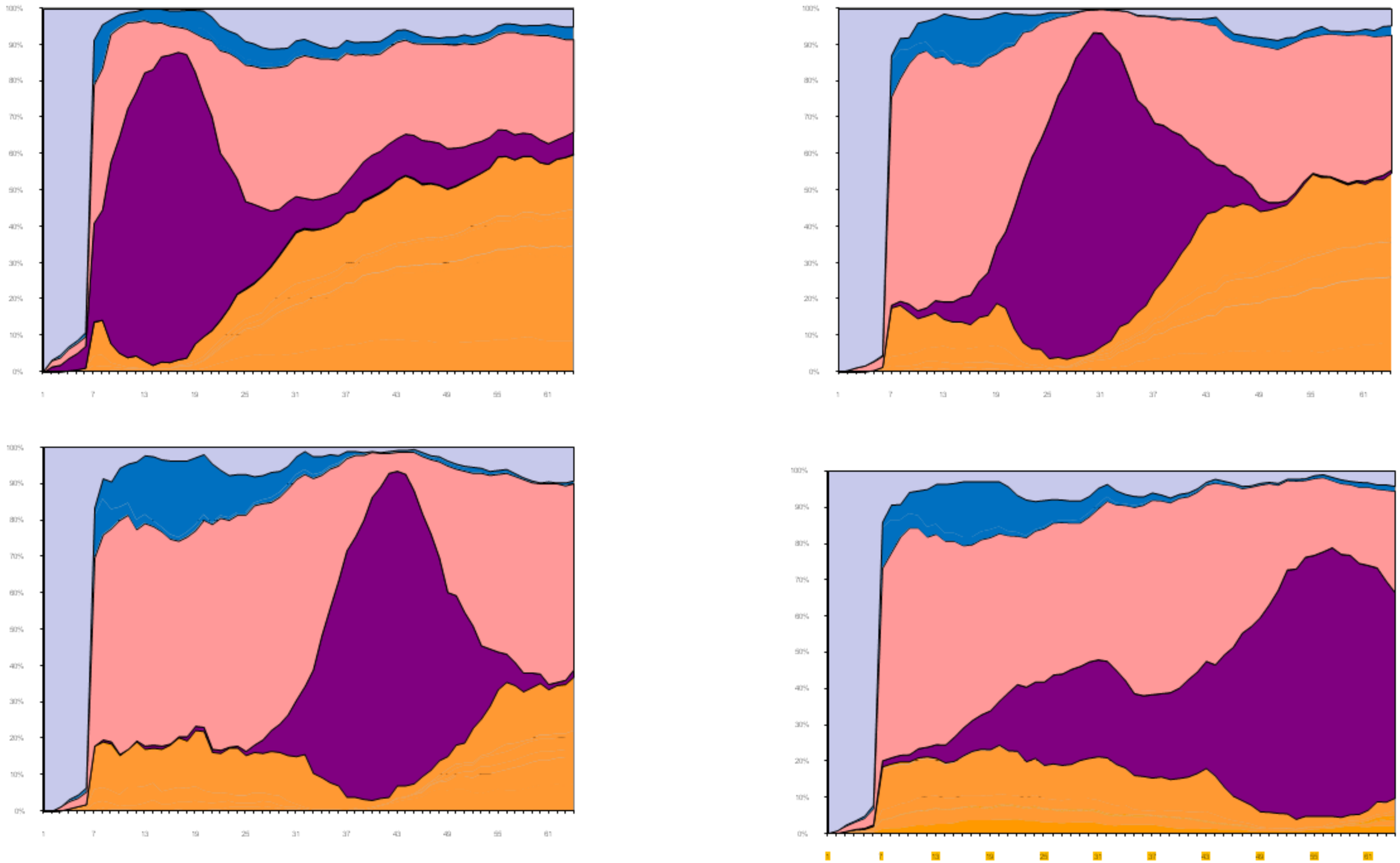
HAC of 2400 pathways (50 factors) → 9 clusters



One among 9 clusters: youngsters who went through a CES (employment scheme)



Split into 4 clusters: time of occurence



5- Conclusion - discussion

- One cluster analysis for one question
- The classification used to distinguish between states (events) has a decisive effect on the result
- Also the way data are transformed (monthly calendar, set of periods, indicators …)
- If possible Cluster analysis and Factor analysis (complementary methods)
- How many clusters?

Thank you for your attention

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